

remarks on unification

richard waldinger

herbrand award

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[joint work with zohar manna]

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jacques herbrand

- studied with emmy noether, von neumann, artin.
- logic was not highly regarded.
- proved consistency of mathematics.
- reconciled with godel's finding.
- died in mountain climbing accident at age 23!



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herbrand and unification

- robinson credited herbrand with unification.
- herbrand treatment analogous to solving sets of simultaneous equations.
- did not have notion of substitution as object.
- no idea of composition of substitution.

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synthesis of unification

- find most general unifier if one exists.
- otherwise, indicate non-unifiable.
 - expression or tuples of expressions.

boyer moore generalization

- “inventor’s paradox” [Polya]
- to prove a theorem, strengthen it to get the benefit of a stronger induction hypothesis.

strengthening of unification theorem

- termination proof does not go through.
- must specify that algorithm yields "idempotent" unifier.

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idempotent unifiers

- $\theta \circ \theta = \theta$
- no variable on left and right side.
- $\{x \leftarrow f(y)\}$ is idempotent
- $\{x \leftarrow f(z), y \leftarrow z, z \leftarrow x\}$ is not.

recursive calls in unification: function application

- $\text{unify}(f(s_1, \dots, s_n), f(t_1, \dots, t_n)) =$
 $\text{unify}(\langle s_1, \dots, s_n \rangle, \langle t_1, \dots, t_n \rangle)$
- reduces size of inputs
- does not increase set of distinct variables.

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recursive calls in unification: lists

- $\text{unify}(\langle s_1, \dots, s_n \rangle \ \langle t_1, \dots, t_n \rangle) =$
let θ be $\text{unify}(s_1, t_1)$ in
 $\text{unify}(\langle s_2\theta, \dots, s_n\theta \rangle, \langle t_2\theta, \dots, t_n\theta \rangle)$
- arguments may get bigger
- if θ is idempotent, set of distinct variables is reduced.

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termination of unification

- one recursive call reduces size of inputs but may leave variable set the same.
- another recursive call may increase size but reduces set of variables in the arguments.
- lexicographic ordering:
 - $\langle \text{variable set, size of expression} \rangle$

proposal: discovery during synthesis

- discover generalization.
- discover well-founded relation.

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synthesis of unification still unfinished

- larry paulson
- alessandro armando et al
- daniele nardi
- lars-henrik eriksson

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herbrand's notion still useful

- alberto martelli and ugo montanari used systems of equations.
- notion used in e.g. constraint logic programming.