

[My thesis advisor was Raymond Smullyan. He likes to begin talks with, "Before I speak, there's something I'd like to say." Well, before I speak, there's something I'd like to say. When, to my surprise, I was told about the Herbrand Award, I decided that for probably once in my life I'd make a speech. This became easier when I realized that a speech is just a talk without slides. I even wrote the speech out, and this is what I'm going to read to you now.]

I want to thank CADE for this wonderful and unexpected award. I am deeply grateful to be honored in this way.

As it happens, almost simultaneously with this award I am retiring from teaching. Not from research, but from regular, day to day teaching. I've been at it for 44 years now. There's a story about a 16 year old boy who was asked what he would want, if he could have anything whatsoever. His answer: "I'd like to be 12 again, but knowing what I know now." It's a good answer; it embodies part of the human condition. Indeed, any two years n and k , with $n < k$ would work just as well.

We here are involved with automated deduction. Deduction is in a logic. I want to express some very general thoughts about our choices of logics. I have nothing profound to say, nothing you probably haven't considered already. But I'm going to say it anyway.

In the nineteenth and early twentieth centuries philosophy often came in big, comprehensive systems. Fichte, Hegel, Schopenhauer, even Marx and Freud, and others. Perhaps the post-modernists can be counted as late outliers in this tradition. Big scale philosophers aspired to a complete world view. In the popular mind this approach was simply identified with philosophy. In 1940, Richard Rodgers and Lorenz Hart wrote a Broadway Musical called "Pal Joey," perhaps their best. In it one character, based on the exotic dancer Gypsy Rose Lee, sings a song that contains the lines, "I was reading Schopenhauer last

night, And I think that Schopenhauer was right.” The assumption was that a Broadway audience would at least know a little of what Schopenhauer was about—at least had heard the name.

Big philosophy is no longer in fashion. For one thing, it’s hard to know when you’re right. Late twentieth century and twenty-first century philosophy, by and large, is much more fragmented. Papers and books are on specialized topics and few, if any, try to be universal. For instance, a few years ago I heard Saul Kripke give a very nice, witty, and informative lecture on the word “the.” [In fact, there is much to be said about the word “the,” but this is certainly at the opposite extreme from the big approaches of former times.

Formal logic follows the same curve as philosophy. In the late nineteenth and early twentieth centuries Frege, and especially Russell and Whitehead, created big logics. For mathematics, indeed for the sciences generally, they created large, all encompassing formal systems. Richard Montague is a late twentieth century outlier in this tradition, having created a kind of universal formal logic that could embrace intensional as well as extensional concepts.

But from the later period of the twentieth century to today, formal logic has fragmented. There are now many, many small logics, decidable if at all possible, each designed to treat a narrow area of human thought and its applications. You all know many items in the list. Modal logics. Epistemic Logics. Temporal logics. Many-valued logics. Many-valued Modal logics. Non-Monotonic logics, Substructural logics. Paraconsistent Logics. Dynamic Logics. Logics of Communication. Game Logics. Fuzzy Logics. Justification Logics. This is hardly complete. It’s a long list, and growing steadily. And those I mentioned are not single logics, but families of logics. Each family is devoted to a narrow aspect of reasoning and a specific intended subject matter, which is further narrowed down by a choice of logic within the family.

[I'm not slighting first-order logic. It is central, but in terms of my big/small classification, it's not quite one and not quite the other. It is not decidable so it's not small. But it is surprisingly tractable in a pragmatic sense, so it's not big either. I'm sure we all know the saying, "First-order logic, and then order beer.]

The move to multiple specialized logics is a natural one from the point of those here. Any huge, general, "logic to rule them all" is complex, difficult to work with, impossible to automate. [And here a nod to FOL, which succeeds surprisingly well.] With a divide and conquer strategy we fragment the world of logic into more docile bits. At least to some extent. As we all know, proof methods that work splendidly for one general variety of logic may be inapplicable to another. Even decidable logics can have decision procedures that render the fact of decidability essentially useless. Still, progress is being made. And we are certainly—well, probably—moving closer to the way we reason ourselves.

When we do mathematics, what is the role of a big system such as *Principia Mathematica*, or Martin-Löf, or ZF? It tells us the minimum general assumptions that are needed for what we do. That's important, but in mathematical practice we don't actually start from there. We work in, say, group theory, or projective geometry, or topos theory, or functional analysis, and we start with assumptions and methods of reasoning peculiar to that area, plus some general machinery which we grab as needed. We don't begin by asking, "what reasoning machinery must we use?" Instead we decide what we are reasoning about and pick the machinery accordingly. If this is so in mathematics, how much more so in everyday life where, all appearances to the contrary, we do apply reason.

The fundamental question we must address, in the face of this fragmentation of logics, is how to put it all back together. Think of the array of logics we work with as formal representations of ways we

sometimes think. How do we manage to function daily, act coherently, and not explode periodically? Well, perhaps that's a bit too much to ask, but let's try anyway.

We could look for principles that allow logics to be combined, then combine the whole bunch once and for all. Of course, there is already some formal machinery for combining logics, but its primary use is to combine small logics into another manageable small logic. I don't think anybody has pushed things much beyond that. As such it is a useful tool for the creation of small logics. But combine them all somehow? The outcome must be yet another big logic of the classic kind, but more complicated than anything previously considered. This does not seem to be a promising direction at all. And it does not seem to be how we behave ourselves, as what L. Frank Baum [the author of "The Wizard of Oz"] called, rather gruesomely, "meat men," [contrasting with his "tin man."]

We seem to operate pragmatically in the presence of multiple ways of reasoning. We somehow evaluate what method is most appropriate and go with it. We ignore ways of reasoning that seem, on their face, useless to the particular job. I have a new grandchild, and he is in the process of learning how to do this. Perhaps he will tell me about it someday, for I have forgotten the details. More likely he will not because it is not something that rises to the level of conscious thought. It is a kind of subroutine in our operating systems, routing problems to appropriate solution mechanisms. Or perhaps this is simply a plausible answer that my appropriate solution mechanism has come up with.

However it is done, we still have two basic problems for our understanding. What do we do when different, but still appropriate, reasoning mechanisms produce different results? The second problem is a special case, what do we do when a reasoning mechanism produces a result that is discovered to be false? (This is a special case because the

world is a kind of logic, with experiments as searches for counter-examples.) Again, I don't know how we handle this. Perhaps each of us has different ways of coping with these problems. I don't believe we have one superior internal system that judges—this would be the one universal system again. It seems more likely that all this remains a local matter; somehow two conflicting results fight it out. Our overall rational behavior, such as it is, is the outcome of all this constant divide-and-conquer, strongest solution prevails battleground. At least, it's the best I can come up with.

But it does suggest a challenge for the coming generations of formal logicians and researchers in automated deduction. We now have many reasoning systems, of varying degrees of strength, of diverse areas of applicability. How do we fit them together? And it should not be by our brute force. I don't think we are up to the task of deciding, once and for all, what problems are appropriate for what mechanisms and who prevails under conflicts. We need to devise a learning strategy that can allot tasks, adjudicate conflicts, and accommodate the addition of new ways of reasoning. This itself would not be a logic, but something more akin to an operating system that learns by experience. One might begin with a few well-understood formal logics, and see what is needed to combine them, not formally, but pragmatically, operationally. They should remain separate, but things should be managed so that each submitted problem is, somehow, routed to the more appropriate logic, or logics if that is appropriate, conflicts between logics and between them and the world are somehow resolved, and the management system learns from its mistakes. What is learned is not logic, but how to choose which logic.

What sort of input would be required? At its best, it would be the description of a problem that somehow requires reasoning, stated in natural language. And what sort of output? Either a proof, or a description of a counter-model, ideally in natural language. Below the surface the right formal logic would be selected, or battled for, and its

output made appropriate use of. In fact there is a plausible model for what is needed: the recent *Watson* program, designed to play the TV show *Jeopardy*. At the Turing 100 conference, immediately preceding this one in Manchester, David Ferrucci gave a talk about the development of *Watson*. The ideas and architecture are more-or-less right for the task I outlined, and he said the design features are migrating to simpler computer systems. Perhaps soon such machinery will be available to this community. Perhaps soon enough.

Judea Pearl says, "... we utilize knowledge about irrelevance. We decompose problems into chunks that are only loosely connected." Yes, but when we reason in mathematics relevance plays a role that is perhaps as big as irrelevance. This may carry over to the rather specialized task of selecting logics. I'm not talking about artificial intelligence generally, but about something much narrower. This is still a big job, since we already have many, many formal, automated logics. For that matter, it is a big job with just two, which is where one might begin. It's not exactly logic. It's logic management. I have no idea how to proceed. But it is worth thinking about.

Thank you for your attention.